

Resonator-assisted entangling gate for singlet-triplet spin qubits in nanowire double quantum dots

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We propose a resonator-assisted entangling gate for spin qubits with high fidelity. Each spin qubit corresponds to two electrons in a nanowire double quantum dot, with the singlet and one of the triplets as the logical qubit states. The gate is effected by virtual charge dipole transitions. We include noise in our model to show feasibility of the scheme under current experimental conditions.

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Quantum computing enables some computational problems to be solved faster than would ever be possible with a classical computer [1] and exponentially speeds up solutions to other problems over the best known classical algorithms [2]. Of the promising technologies for quantum computing, solid-state implementations such as spin qubits in quantum dots (QDs) [3] and bulk silicon [4] and charge qubits in bulk silicon [5] and in superconducting Josephson junctions [6] are especially attractive because of stability and expected scalability of solid-state systems. One of the most promising qubits in QDs corresponds to a pair of electrons in a pair of closely-spaced quantum dots [7–9] such that the logical qubit state $|0\rangle$ is the spin singlet and $|1\rangle$ is one of the spin triplets. Universal quantum computing is possible if general single-qubit gates and one entangling two-qubit gate can be performed.

Here we develop a two-qubit gate for semiconductor quantum computation based on two-spin states in double quantum dots (DQDs) realized in the nanowires (NWs) coupled to a superconducting stripline resonator (SSR). Compared to the previous proposals which make use of single dots or DQDs defined by a two-dimensional electron gas (2DEG) [10–13], our proposal is more realistic for implementation. It would be difficult to realize a DQD in a planar resonator with lateral dots, shaped in a 2DEG by surface gates. Because it is difficult to prevent absorption of microwaves in the 2DEG unless one can make the electric field non-zero only in the DQD region, which is not realistic experimentally yet. A more realistic implementation can be done with NWs as it shows in this work. Very recently, a spin dynamics in InAs NW QDs coupled to a transmission line via the spin-orbit interaction has been proposed [14]. In this work, we propose another mechanism to achieve an entangling gate between spins inside a SSR, namely via resonator-assisted interaction which leads to an efficient coupling between the resonator photon and the effective electric dipole of DQD and thus eventually entangles the singlet and one of the triplet spin states.

We consider the system with two electrons located in adjacent QDs inside a semiconductor NW, coupled via tunneling shown in Fig. 1. One of the dots is capacitively

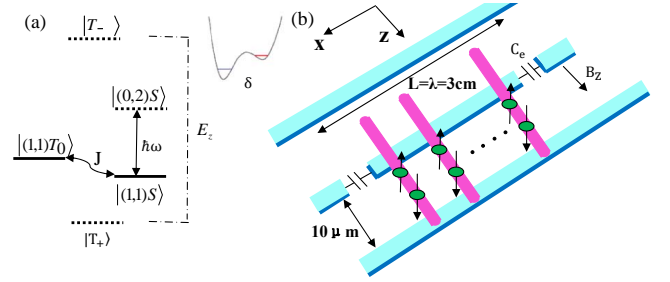


FIG. 1: (a) Energy level diagram showing the $(0, 2)$ and $(1, 1)$ singlets, the three $(1, 1)$ triplets and the qubit states $|(1, 1)S\rangle$ and $|(1, 1)T_0\rangle$ with the energy gap J (the exchange energy). Schematic of the double-well potential with an energy offset δ provided by the external electric field. (b) Schematic of NW DQDs capacitively coupled to the SSR. The coupling can be switched on and off via the external electric field. The DQD confinement can be achieved by barrier materials or by external gates (not shown).

coupled to a SSR. With an external magnetic field B_z along the z axis, the spin aligned states $|T_+\rangle = |\uparrow\uparrow\rangle$ and $|T_-\rangle = |\downarrow\downarrow\rangle$, and the spin-anti-aligned states $|(1, 1)T_0\rangle = (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$ and $|(1, 1)S\rangle = (|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)/\sqrt{2}$ have energy gaps due to the Zeeman splitting shown in Fig. 1(a). The notation (n_L, n_R) labels the number of electrons in the left and right QDs of a pair. Considering a three-level system, we choose the singlet state and one of the triplet states as our qubit states:

$$|0\rangle \equiv |(1, 1)S\rangle; \quad |1\rangle \equiv |(1, 1)T_0\rangle, \quad (1a)$$

and the doubly occupied state as an ancillary state

$$|a\rangle \equiv |(0, 2)S\rangle. \quad (1b)$$

The DQD can be described by an extended Hubbard Hamiltonian $\hat{H} = (E_{os} + \mu) \sum_{i,\sigma} \hat{n}_{i,\sigma} - T \sum_{\sigma} (\hat{a}_{L,\sigma}^\dagger \hat{a}_{R,\sigma} + \text{hc}) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + W \sum_{\sigma,\sigma'} \hat{n}_{L,\sigma} \hat{n}_{R,\sigma'} + \delta \sum_{\sigma} (\hat{n}_{L,\sigma} - \hat{n}_{R,\sigma})$ for $\hat{a}_{i,\sigma}$ ($\hat{a}_{i,\sigma}^\dagger$) annihilating (creating) an electron in a QD $i \in \{L, R\}$ with spin $\sigma \in \{\uparrow, \downarrow\}$, $\hat{n}_{i,\sigma} = \hat{a}_{i,\sigma}^\dagger \hat{a}_{i,\sigma}$ a number operator, and δ an energy offset yielded by the external electric field. The first term corresponds

to on-site energy E_{os} plus site-dependent field-induced corrections μ . The second term accounts for $i \leftrightarrow j$ electron tunneling with rate T . The third term is the on-site charging cost U to put two electrons with opposite spin in the same dot, and the fourth term corresponds to inter-site Coulomb repulsion. In the basis $\{|0\rangle, |a\rangle\}$, the Hamiltonian can be reduced as

$$\hat{H}_d = -\delta |a\rangle \langle a| + T |0\rangle \langle a| + \text{hc.} \quad (2)$$

With the energy offset δ , degenerate perturbation theory in the tunneling T reveals an avoided crossing at this balanced point which occurs at the left-most avoided crossing between $|0\rangle$ and $|a\rangle$ with an energy gap $\omega = \sqrt{\delta^2 + 4T^2}$.

The essential idea is to use an effective electric dipole moment associated with singlet states $|0\rangle$ and $|a\rangle$ of a NW DQD coupled to the oscillating voltage associated with a SSR shown in Fig. 1(b). Whereas the qubit state $|1\rangle$ is decoupled to the SSR due to the large energy gap J . We consider a SSR with the length L , the capacitance per unit length C_0 and the characteristic impedance Z_0 . A capacitive coupling C_c between the NW DQD and SSR causes the electron charge state to interact with excitations in the transmission line. We assume that the dot is much smaller than the wavelength of the resonator excitation, so the interaction strength can be derived from the electrostatic potential energy of the system $\hat{H}_{\text{int}} = e\hat{V}v |(0, 2)S\rangle \langle (0, 2)S|$ with e the electron charge, $\hat{V} = \sum_n \sqrt{\frac{\hbar\omega_n}{LC_0}}(\hat{c}_n + \hat{c}_n^\dagger)$ the voltage on the SSR near the left dots, \hat{c}_n (\hat{c}_n^\dagger) the creation (annihilation) operator for the SSR mode $k_n = [(n+1)\pi]/L$, $v = C_c/C_{\text{tot}}$, and C_{tot} the total capacitance of the DQD. The fundamental mode frequency of the SSR is $\omega_0 = \pi/LZ_0C_0 \approx \omega$. The SSR is coupled to a capacitor C_e for writing and reading the signals. Neglecting the higher modes and working in the rotating frame with the rotating wave approximation, we obtain an effective interaction Hamiltonian as

$$\hat{H}_{\text{int}} = g(\hat{c}|a\rangle \langle 0| + \text{hc}) \quad (3)$$

with the effective coupling coefficient

$$g = \frac{1}{2}ev \frac{1}{LC_0} \sqrt{\frac{\pi}{Z_0\hbar}} \sin 2\vartheta \quad (4)$$

with $\vartheta = \frac{1}{2} \tan^{-1}(\frac{2T}{\delta})$.

The interaction between the SSR and qubit states is switchable via tuning the electric field. In the case of the energy offset yielded by the electric field $\delta \approx 0$, we obtain the maximum value of the coupling between the SSR and qubit states. Whereas $\delta \gg T$, ϑ tends to 0, the interaction is switched off.

We consider there are two NW DQDs coupled to the SSR. If both of the DQDs are in the state $|1\rangle$, the incoming pulses which are resonant with the bare resonator mode and performed in the limit with $\tau \gg 1/\kappa$ (here τ is the pulse duration and κ is the resonator decay rate)

are resonantly reflected by the bare resonator mode with a flipped global phase of π from the standard quantum optics calculation [15, 16]. For the other three cases including that the DQDs are in the states $|00\rangle$, $|01\rangle$ and $|10\rangle$ the frequency of the dressed resonator mode is significantly detuned from the frequency of the incoming pulses and the frequency shift has a magnitude comparable with g . Thus the SSR functions as a mirror and the shape and global phase of the reflected pulses remain unchanged.

We now go to a detailed description of the resonator-assisted interaction with the input field in an even coherent state $|\alpha\rangle_- = N_- (|\alpha\rangle - |-\alpha\rangle)$, where N_- is normalization constant and $|\alpha\rangle$ is a coherent state. Recently, this novel state of light has been generated and characterized by a non-positive Wigner function experimentally [17]. The SSR output \hat{c}_{out} is connected with the resonator mode \hat{c} via the following relations

$$\dot{\hat{c}}(t) = -i \left[\hat{c}(t), \hat{H} \right] - (i\Delta + \frac{\kappa}{2})\hat{c}(t) - \sqrt{\kappa}\hat{c}_{\text{in}}(t); \quad (5)$$

$$\hat{c}_{\text{out}}(t) = \hat{c}_{\text{in}}(t) + \sqrt{\kappa}\hat{c}(t). \quad (6)$$

Here Δ denotes the detuning of the resonator field mode $\hat{c}(t)$ from the input pulse $\hat{c}_{\text{in}}(t)$ with the standard communication relation $[\hat{c}_{\text{in}}(t), \hat{c}_{\text{in}}(t')] = \delta(t - t')$. We get the following results. If both DQDs are in the state $|1\rangle$, the Hamiltonian \hat{H} is not active. Based on Eqs. (5) and (6) we obtain $\hat{c}_{\text{out}}^{11}(t) = -\hat{c}_{\text{in}}^{11}(t)$ if the resonant interaction satisfies $\Delta = 0$ and the input pulse shape changes slowly with time t compared with the resonator decay rate κ . That means if the state of both DQDs is in $|11\rangle$, the output field acquires a phase π after the interaction. In other input cases, however, there are effective detunings of the dressed resonator mode from the input pulse $\pm\sqrt{2}g$ for $|00\rangle$ and $\pm g$ for $|01\rangle$ ($|10\rangle$) respectively and the input-output equations $\dot{\hat{c}}_{\text{out}}^{mn}(t) = \xi_{mn}\hat{c}_{\text{in}}^{mn}(t)$ ($m, n = 0, 1$) where $\xi_{11} = -1$, $\xi_{00} = (8s - 1)/(8s + 1)$ and $\xi_{01} = \xi_{10} = (4s - 1)/(4s + 1)$ with $s = g^2T_1/\kappa$ (T_1 is charge relaxation time). If $s \gg 1$ is satisfied, ξ_{00} , ξ_{01} and ξ_{10} tend to 1. Thus for an arbitrary initial state, after the resonator-assisted interaction and tracing out the resonator field, we implement a controlled phase flip (CPF) gate $e^{i\pi|11\rangle\langle 11|}$ on the spin states of the two DQDs and the gate time is about $t_{\text{cpf}} \sim \tau$ (τ is the pulse duration).

Now we analyze the feasibility of the proposal with the elongated QDs oriented along the NW. QDs have been realized within NWs in various material systems [18]. A realization of DQDs defined using local gates to electrostatically deplete InAs NWs grown by chemical beam epitaxy was reported [19]. The quantum-mechanical tunneling T between the two QDs is about $0 - 150\mu\text{eV}$ [19]. Thus at the optimal point $\delta \approx 0$ where the coupling is strongest, the energy gap between the singlets is about $\omega \sim 2T \simeq 0 - 72\text{GHz}$. The SSR can be fabricated with existing lithography techniques [20]. The dots can be placed within the SSR formed by the transmission line

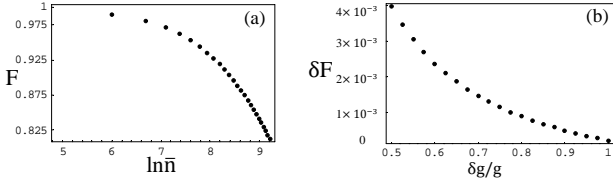


FIG. 2: (a) The fidelity of the CPF gate versus the mean photon number of the input pulse with the pulse duration $\tau = 10/\kappa$, and (b) it changes with $\delta g/g$. Here we choose the realistic parameters $(g, \kappa, 1/T_1)/2\pi = (120, 100, 1)$ MHz.

to strongly suppress the spontaneous emission. A small-diameter ($d \sim 65$ nm), long-length ($l \sim 270$ nm) and $g^* = -13$ [21] InAs NW is positioned perpendicularly to the transmission line and containing DQDs that are elongated along the NW shown in Fig. 1(b). To prevent a current flow, the NW and the transmission line need to be separated by some insulating coating material obtained for example by atomic layer deposition. We assume that the SSR is 3 cm long and $10 \mu\text{m}$ wide, $Z_0 = 50 \Omega$, which implies for the fundamental mode $\omega_0 = 2\pi \times 10$ GHz. The external magnetic field along the z axis is about $B_z = 1$ T to make sure the energy splitting $E_z = g^* \mu_B B_z$ between the two triplet states $|T_{\pm}\rangle$ is larger than $\hbar\omega$. In practice, the careful fabrication permits a strong coupling capacitance, with $v \approx 0.2$ [19], so that the coupling coefficient $g \sim 2\pi \times 120$ MHz is achievable due to the numerical estimations in Eq. (4). The frequency ω_0 and coupling coefficient g can be tuned via LC_0 . In order to implement gates on a fast time scale, the system works in a weak coupling regime. In the bad cavity limit, we have $g \sim \kappa$, where $\kappa = \omega_0/Q \approx 2\pi \times 100$ MHz with the quality factor of the SSR $Q = 100$ [22]. Considering the effect of photon assisted tunneling in our system, which is harmful because it destroys the qubit by lifting spin-blockade. To avoid this, one needs to close enough the tunneling barriers to the leads.

We address now the issue of relaxation and decoherence of our system. There are three types of contributions to the relaxation processes, one arising from the finite decay rate of the SSR, one from the intrinsic decoherence of the spin states, and the other one from the charge-based dephasing and relaxation which occur during the gate operation involving the electric dipole between $|a\rangle$ and $|0\rangle$.

For the charge relaxation time T_1 , the decay is caused by coupling qubits to a phonon bath. With the spin-boson model, the perturbation theory gives an overall error rate from the relaxation and incoherent excitation, with which one can estimate the relaxation time $T_1 \sim 1 \mu\text{s}$ [12]. That is studied in great details for the GaAs QDs in 2DEG and a similar rate is expected for NW QDs.

The charge dephasing T_2 rises from variations of the energy offset $\delta(t) = \delta + \varepsilon(t)$ with $\langle \varepsilon(t)\varepsilon(t') \rangle = \int d\omega S(\omega) e^{i\omega(t-t')}$, which is caused by the low frequency fluctuations of the electric field. The gate bias of the

qubit drifts randomly when an electron tunnels between the metallic electrode. At the zero derivative point, compared to a bare dephasing time $T_b = 1/\sqrt{\int d\omega S(\omega)}$, the charge dephasing is $T_2 \sim \omega T_b^2$ near the optimal point $\delta = 0$. The bare dephasing time $T_b \sim 1$ ns was observed in [23]. Then the charge dephasing is about $T_2 \sim 10 - 100$ ns. Using quantum control techniques, such as better high- and low-frequency filtering of electronic noise, T_b exceeding $1 \mu\text{s}$ was observed in 2DEG [7] (we assume a similar result for the present case), which suppresses the charge dephasing.

The hyperfine interactions with the host nuclei cause nuclear spin-related dephasing T_2^* . The hyperfine field can be treated as a static quantity, because the evolution of the random hyperfine field is several orders slower than the electron spin dephasing. In the operating point, the most major decoherence due to the hyperfine field is the dephasing between the singlet state $|0\rangle$ and one of the triplet states $|1\rangle$. By suppressing nuclear spin fluctuations, the spin dephasing time can be obtained by quasi-static approximation as $T_2^* = 1/g\mu_B \langle \Delta B_n^z \rangle_{\text{rms}}$, where ΔB_n^z is the nuclear hyperfine gradient field between two coupled dots and rms means a root-mean-square time-ensemble average. A measurement of the spin dephasing time $T_2^* \sim 4$ ns was demonstrated in [14] and we expect that coherently driving the qubit will prolong the T_2^* time up to $1 \mu\text{s}$ and with echo up to $10 \mu\text{s}$ [7].

The quality factor Q of the SSR in the microwave domain can be achieved up to 10^6 [20]. In practice, the local external magnetic field $B_z = 1$ T reduces the limit of Q [22]. However, in our proposal the low Q SSR is good for implementing an entangling gate with short operation time. The dissipation of the SSR κ leads the decay time about 10 ns, which will cause photon loss. Due to the photon loss, the amplitude of the output field α_{mn} for $m, n = 0, 1$ representing the different initial states of the DQDs is probably different from the input amplitude α and usually $|\alpha_{mn}| < |\alpha|$. The fundamental source of photon loss in the SSR can be qualified by the parameter $\eta = 1 - \min\{|\alpha_{mn}/\alpha|^2\} \propto \kappa/g^2 T_1$.

Except for the photon loss, the function distortion is another reason which causes the noise. The input to the SSR is a coherent field which can be described as $|\alpha\rangle_{\text{in}} = e^{\alpha \int_0^\tau f_{\text{in}}(t) \hat{c}_{\text{in}}^\dagger(t) dt} |vac\rangle$, where $f_{\text{in}}(t)$ describes the input pulse shape with normalization $\int_0^\tau |f_{\text{in}}(t)|^2 dt = 1$ with the pulse duration τ . We calculate the output pulse shapes $f_{\text{out}}^{mn}(t)$ from the expectation value of the input-output equation (6) $\alpha_{mn} f_{\text{out}}^{mn}(t) = \alpha f_{\text{in}}(t) + \sqrt{\kappa} \langle \hat{c}(t) \rangle$. The expectation value of the resonator mode $\hat{c}(t)$ can be obtained by solving the corresponding master equation

$$\begin{aligned} \dot{\rho} = & -i[\hat{H}_{\text{eff}}, \rho] + \frac{\kappa}{2}(2\hat{c}\rho\hat{c}^\dagger - \hat{c}^\dagger\hat{c}\rho - \rho\hat{c}^\dagger\hat{c}) \\ & + \frac{1}{2T_1}(2\hat{\sigma}_-\rho\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\rho - \rho\hat{\sigma}_+\hat{\sigma}_-) \end{aligned} \quad (7)$$

with $\hat{\sigma}_+ = |a\rangle\langle 0|$, $\hat{\sigma}_- = |0\rangle\langle a|$ and the effective Hamiltonian $\hat{H}_{\text{eff}} = g\hat{\sigma}_+\hat{c} + i\sqrt{\kappa}\langle\hat{c}_{\text{in}}(t)\rangle\hat{c} + \text{hc}$. By solving the

master equation we obtain $\langle \hat{c}(t) \rangle = \text{tr}(\rho \hat{c}(t))$. Then we can determine the output amplitude α_{mn} and the corresponding pulse shape $f_{\text{out}}^{mn}(t)$ and define the mismatching of the input and output pulses for the initial state $|mn\rangle$ as $\epsilon_{mn} = 1 - \int \int f_{\text{in}}^*(t') f_{\text{out}}^{mn}(t) dt dt'$.

Now we analyze the fidelity of the CFP gate under the influence of some practical sources of noise. For the initial state of the system $|\Phi\rangle_{\text{in}}$ with the input pulse $|\phi\rangle_{\text{in}} \propto [e^{\alpha \int_0^\tau f_{\text{in}}(t) \hat{c}_{\text{in}}^\dagger(t) dt} - e^{-\alpha \int_0^\tau f_{\text{in}}(t) \hat{c}_{\text{in}}^\dagger(t) dt}] |vac\rangle$ the

input pulse corresponding to the double-dot state $|mn\rangle$. By applying a CFP gate, the output state of the system can be written as $|\Phi\rangle_{\text{out}}$ with the output pulse $|\phi\rangle_{\text{out}}^{mn}$ with a shape $f_{\text{out}}^{mn}(t)$ and amplitude α_{mn} different from the input: $|\phi\rangle_{\text{out}}^{mn} \propto [e^{\alpha_{mn} \int_0^\tau f_{\text{out}}^{mn}(t) \hat{c}_{\text{out}}^\dagger(t) dt} - e^{-\alpha_{mn} \int_0^\tau f_{\text{out}}^{mn}(t) \hat{c}_{\text{out}}^\dagger(t) dt}] |vac\rangle$. Then the fidelity can be defined as

$$F = \langle \Phi_{\text{ID}} | \rho(t_{\text{cpf}}) | \Phi_{\text{ID}} \rangle \quad (8)$$

$$= \left| \left\{ \sum_{mn=00,01,10} e^{-\frac{1}{2}|\alpha|^2[(1-\epsilon_{mn})^2 + (1-\eta_{mn}) - 2\xi_{mn}\sqrt{1-\eta_{mn}}(1-\epsilon_{mn})]} + e^{-\frac{1}{2}|\alpha|^2[(1-\epsilon_{11})^2 + (1-\eta_{11}) + 2\xi_{11}\sqrt{1-\eta_{11}}(1-\epsilon_{11})]} \right\} / 4 \right|^2,$$

where the ideal output state is $|\Phi_{\text{ID}}\rangle = e^{i\pi|11\rangle\langle 11|} |\Phi\rangle_{\text{in}}$.

We investigate the fidelity under typical experimental configurations and it is shown in Fig. 2 as a function of the mean photon number of the input state with the realistic parameters $(g, \kappa, 1/T_1)/2\pi = (120, 100, 1)\text{MHz}$. In the calculation, we have assumed a Gaussian shape for the input pulse with $f_{\text{in}}(t) \propto e^{[-(t-\tau/2)^2/(\tau/5)^2]}$ with the pulse duration $\tau = 10/\kappa$. We obtain a high fidelity up to 0.99 for these parameters and the coherent input pulse with a remarkable amplitude $\alpha \sim 20$. Furthermore, the fidelity F is insensitive to the variation of the coupling coefficient g caused by the fluctuations in the DQD positions and the energy gap δ . The change of the fidelity δF is about 10^{-3} for g varying to $g/2$. Furthermore, we can estimate the time scaling for the CPF gate operation $t_{\text{cpf}} \sim \tau = 100\text{ns}$ with fidelity 0.99, which is shorter compared to the decoherence time.

In summary, we propose a resonator-assisted entangling gate for singlet-triplet spin qubits in DQDs, which

exploits virtual charge-qubit transitions for double-dot pairs capacitively coupled to a SSR. Because of the switchable coupling between the double-dot pairs and the SSR, we can apply this entangling gate on any two qubits without affecting others, which is not trivial for implementing scalable quantum computing and generating large entangled states. The fidelity of this gate is studied including all kinds of major decoherence, with promising results for reasonably achievable experimental parameters. The feasibility of this scheme is characterized through exact numerical simulations that incorporate various sources of experiment noise and these results demonstrate the practicality by way of current experimental technologies.

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- [1] L. Grover, Phys. Rev. Lett. **79**, 325 (1997).
 - [2] P. W. Shor, Proc. 35th Annual Symp. on Found. of Comp. Sci. (Los Alamitos, CA: IEEE Computer Society Press, 1994), pp 124.
 - [3] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
 - [4] B. E. Kane, Nature **393**, 133 (1998).
 - [5] S. E. S. Andresen et al., Nanolett. **7**, 2000 (2007).
 - [6] A. Wallraff et al., Phys. Rev. Lett. **95**, 060501 (2005).
 - [7] J. R. Petta et al., Science **309**, 2180 (2005).
 - [8] A. C. Johnson, Nature **435**, 925 (2005).
 - [9] J. M. Taylor et al., Phys. Rev. B **76**, 035315 (2007).
 - [10] A. Imamoglu et al., Phys. Rev. Lett. **83**, 4204C4207 (1999).
 - [11] G. Burkard, A. Imamoglu, Phys. Rev. B **74**, 041307(R) (2006).
 - [12] J. M. Taylor and M. D. Lukin, arXiv: cond-mat/0605144.
 - [13] Z. R. Lin et al., Phys. Rev. Lett. **101**, 230501 (2008).
 - [14] M. Trif, V. N. Golovach and D. Loss, Phys. Rev. B **77**, 045434 (2008).
 - [15] D. F. Wall and G. J. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994).
 - [16] L. M. Duan and H. J. Kimble, Phys. Rev. Lett. **92**, 127902 (2004); P. Xue and Y. F. Xiao, *ibid* **97**, 140501 (2006).
 - [17] J. S. Neergaard-Nielsen et al., Phys. Rev. Lett. **97**, 083604 (2006).
 - [18] Z. Zhong, Y. Fang, W. Lu and C. Lieber, Nano Lett. **5**, 1143 (2005); S. De Franceschi et al., Appl. Phys. Lett. **83**, 344 (2003); M. T. Björk et al., Nano Lett. **4**, 1621 (2004).
 - [19] C. Fasth, A. Fuhrer, M. T. Björk and L. Samuelson, Nano Lett. **5**, 1487 (2005).
 - [20] A. Wallraff et al., Nature **431**, 162 (2004).

- [21] M. T. Björk et al., Phys. Rev. B **72**, 201307(R) (2005).
- [22] With a magnetic field about 1T the resonators in coplanar waveguides with $Q \sim 10^2 - 10^3$ have already been demonstrated by L. Frunzio et al., Applied Superconductivity, IEEE Transactions on **15**, 860 (2005).
- [23] T. Hayashi et al., Phys. Rev. Lett. **91**, 226804 (2004); J. R. Petta et al., *ibid* **93**, 186802 (2004).